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A HYBRID HEURISTIC APPROACH TO SOLVE MULTI-OBJECTIVE CVRP

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Abstract

This paper proposes a simple hybrid approach that combines Sweep, K-Means, Minimal Spanning Tree and 2-opt algorithm to solve multi-objective capacitated vehicle routing problem. The objective is to minimize the total distance travelled (directly proportional to the transportation costs) and minimize the number of vehicles.The solution is found using cluster first and route second approach. Clusters are initially formed usinga combination of sweep heuristic and K-means clustering. Optimal routes within each cluster are formedusing minimal spanning tree (MST) and 2 opt algorithm.The proposed approach is tested on Christofides benchmark CVRP instances ranging from 50 to 200 customers.

1. INTRODUCTION

The Vehicle Routing Problem (VRP), a generalized case of Traveling Salesman Problem (TSP) introduced by Dantzig and Ramser in 1959, holds a central place in logistics management (both inbound and outbound logistics) and is one of the most widely studied problems in combinatorial optimization. In the initial period solutions generated were using exact algorithms and classical heuristics. Exact solution algorithms however are impracticable for solving vehicle routing instances beyond 50 customers. A few classical heuristics such as sweep, strip, nearest neighbour, 1-petal, 2-petal, minimal spanning tree, Clarke & Wright savings algorithm (1964), are popular due to easy of applicability and ability to generate fairly good solutions. In the past two decades however, much of the research shows the use of meta-heuristics in solving VRP. Although, the best known meta-heuristics developed for the VRP yield better solutions, they also tend to be more time consuming (Laporte, 1999).

Previous classical heuristics forsolving the multi-objective CVRP includesequential savings algorithm (R.H. Mole and S.R. Jameson, 1976), parallel savings algorithm (Laporte, Semet 1999; K. Altinkemer and B. Gavish, 1991), matching based savings heuristics (Desrochers, Verhoog, 1989), 1-petal algorithm (Foster and Ryan, 1976; Ryan, Hjorring and Glover, 1993), 2-petal algorithm (Renaud, Botor and Laporte, 1996). This paper proposes a simple hybrid heuristic that is a combination of sweep heuristicfor partitioning the spatial space (area) into k zones, formation of clusters using k-means algorithm taking initial centroids from each of the k zones, followed with construction of routes for each cluster using minimal spanning tree (MST) and optimization (improvement) of these routes using 2-opt method.The proposed approach is based on combination of simple classical heuristics and is easy to understand and implement in comparison with meta-heuristics.

The rest of paper is organized into various sections as follows: Section 2 explains the formulation of VRP having a dual objective of minimizing the total distance and minimizing the number of vehicles. Section 3 explains the steps of the proposed methodology. Section 4 presents the performance of proposed approach on select benchmark instances and compare results with popular classical heuristics. Section 5 concludes the paper with important findings and scope for further research.

2. FORMULATION OF MATHEMATICAL MODEL

This includes determining decision variables, objective function & constrain

Decision Variables

The Vehicle Routing Problem is a combinatorial problem whose ground set is the edges of a graph G (V, E). The notation used for this problem is as follows:

G = (V, E): the graph representing the vehicle routing network with vertices V= {v0, v1, v2…, vn} and

E = {(vi, vj) | vi, vj V, i<j} is an arc set

V= {v0, v1, v2…,vn} vertex set where v0 represents depotand v1, v2…,vn represent customer locations

n = no. of nodes corresponding to customer locations for each instance (trip or route)

k= no. of pick-up vehicles all having same capacity

m= total no. of trips corresponding to number of vehicles used

Rk= vehicle route k (k = 1,..,m)

dij=distance (proportional to travel time) between vertices vi and vj (non-negative) (I≠j)

qi= demand from node corresponding to customer location vi

Q= maximum capacity of vehicle

$$xijk= \begin{cases} 1, & \text{if vehicle k travels directly from node } v_i \text{ to node } v_j \\ 0, & \text{otherwise} \end{cases}$$

(i, j = 0,1,..,n) (k = 1,.,m)

$$yik= \begin{cases} 1, & \text{if node } v_i \text{ is serviced by vehicle k} \\ 0, & \text{otherwise} \end{cases}$$

(I, = 0,1,..,n) (k = 1,.,m)

Objective Function

$$Min \sum_{i=0}^{n}\sum_{j=0}^{n} d_{ij} x_{ijk} \,\&\, Min\ m$$

(Minimize total distance travelled (transportation costs)

Constraints

$$\sum_{i=0}^{n} q_i y_{ik} \leq Q \qquad (k = 1,2, \dots .m)$$

(Prevent vehicles from carrying loads more than their capacity Q)

$$\sum_{k=1}^{m} y_{ik} = \begin{cases} m, & (i = 0) \\ 1, & (i = 1,2, \dots, n) \end{cases}$$

(Ensures that each vehicle leaves and arrives at the depot exactly once and each customer is served by one and only one vehicle)

$$\sum_{i=0}^{n} x_{io} = m$$

(Ensures that the vehicles leave once at each node and m times at the depot)

$$\sum_{j=0}^{n} x_{oj} = m$$

(Ensures that the vehicles arrive at each node once and leaves the depot m times)

$$\sum_{i=0}^{n} x_{ijk} = y_{jk} (j=0,1,..,n)\ (k=1,\dots,m)$$

(Ensures that the total number of trips is equal to the total number of vehicles)

3. METHODOLOGY

The methodology for solving multi-objective CVRP is explained in the paragraphs below. Figure 2 graphically illustrates the four stages involved in solving the problem with proposed approach.

3.1 Form 'k' Partitions on the Spatial space (Area): Determine the minimum number of vehicles (k) by dividing the total customer demand by vehicle capacity rounded to the next integer. Accordingly, the spatial space (area) is divided into 'k' partitions using sweep algorithm (B. E. Gillett and L. R. Miller, 1974) as shown in step 1 of figure 2.

3.2 Identification of Initial 'k' Centroids: Initial centroids for each partition are calculated as centroid of area corresponding to that partition as shown in step 2 of figure 2. In the classical k-means algorithm initial centroids are either specifically or randomly chosen and may not yield better clusters always. This may affect the solution quality.

3.3 Iterative Modification & Formation of 'k' Clusters Using k-means Algorithm: Figure 1 shows an illustration of K-means algorithm (Jain, A., Murthy, M.N., Flynn, P.J,1999) on a two-dimensional dataset with three clusters. Figure 1(a) shows the nodes to be served. In figure 1(b) three seed points are randomly selected as cluster centroids and nodes are allotted to that cluster whose seed point is nearest to that customer location. Figure 1(c) and 1(d) shows iterations for recalculation of cluster centroids and reallocation of nodes to nearest cluster centroid. Figure 1(e) shows the final clusters obtained by K-means. Similar approach is adopted for formation of clusters in our case.

Customers are allocated to the clusters in descending order of their demand to ensure that all customers fit into minimum number of vehicles.The finally formed clusters may not belong to the initial partitions.The iterative process of cluster formation continues till the specified number of iterations are performed. Step 3 in figure 2 shows the formation of clusters using k-means algorithm for Christofide c-100b VRP instance. In case all customers are not accommodated in 'k' routes, the value of k is incremented by 1. Go to step 1 and form 'k+1'partitions and repeat the procedure.

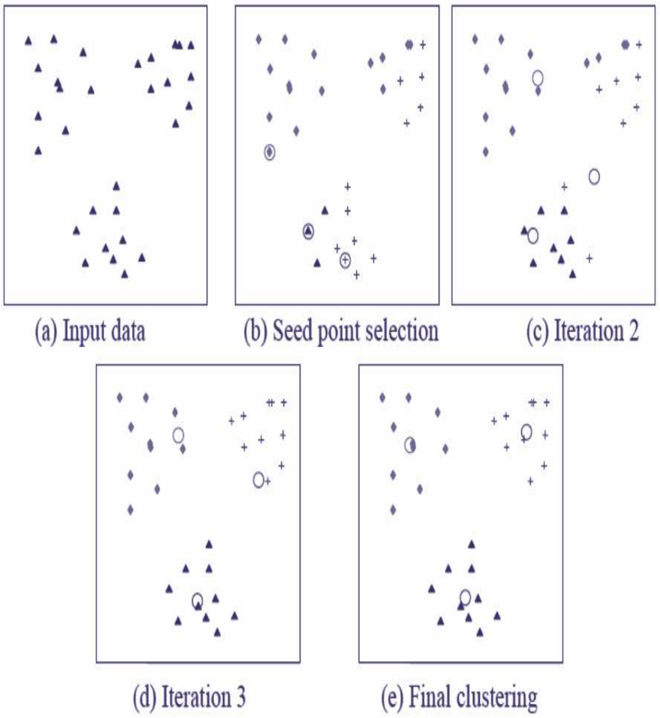

Figure 1: Illustration of K-means algorithm [Jain, A.K (1950)]

3.4Formation of Routes for Each Cluster using MST + 2-opt: The final route for each cluster is constructed using minimal spanning tree (M. Held and R.M. Karp, 1970). 2-opt algorithm is used to make improvements in the formed route.Step 4 in figure 2 shows the routes formed using the proposed approach for all CVRP instances of Christofides.

A brief description of the methodology adopted in form of an algorithm is given below:

Step 1: Calculate minimum number of vehicles 'k' required by dividing total demand with individual vehicle capacity.

Step 2: Divide the spatial space (that accommodates depot and customer locations) into 'k' equal zones using sweep algorithm.

Step 3:Calculation of centroid for each of the individual 'k' zones identified in step 2 above.

Step 4: Formation of initial clusters corresponding to initial centroids calculated in step 3 above.

Step 5: For 1 to n iterations, repeat steps 6 to 8:

Step 6: Modify the centroids and form new 'k' clusters verifying that capacity constraints are not violated and total customer demand is met with 'k' vehicles. If not, go to step 1 and increment value of 'k' by 1.

Step 7: For each of the formed 'k' clusters in step 6 above, form 'k' routes using minimal spanning tree approach.

Step 8: Improve the routes in step 7 above using 2-opt method.

Step 9: Increment 'n'

Step 10: Identify and retain the best solution amongst the 'n' solutions obtained above.

3. PERFORMANCE OF PROPOSED APPROACH ON SELECTED BENCHMARK INSTANCES

The proposed algorithm was tested on all CVRP benchmark instances of Christofides. Table 1 shows the comparison of the proposed approach with well-known classical heuristics. The proposed approach uses minimum number of vehicles compared with other classical heuristics. This is significant when the dual objective of minimum total distance and minimum number of vehicles is to be attained which substantially reduces the fixed costs associated with vehicles. It is observed from table 1 that the proposed method gives better solutions compared with other simple classical heuristics. It is easy to understand and implement and also consumes lesser time compared with metaheuristics.

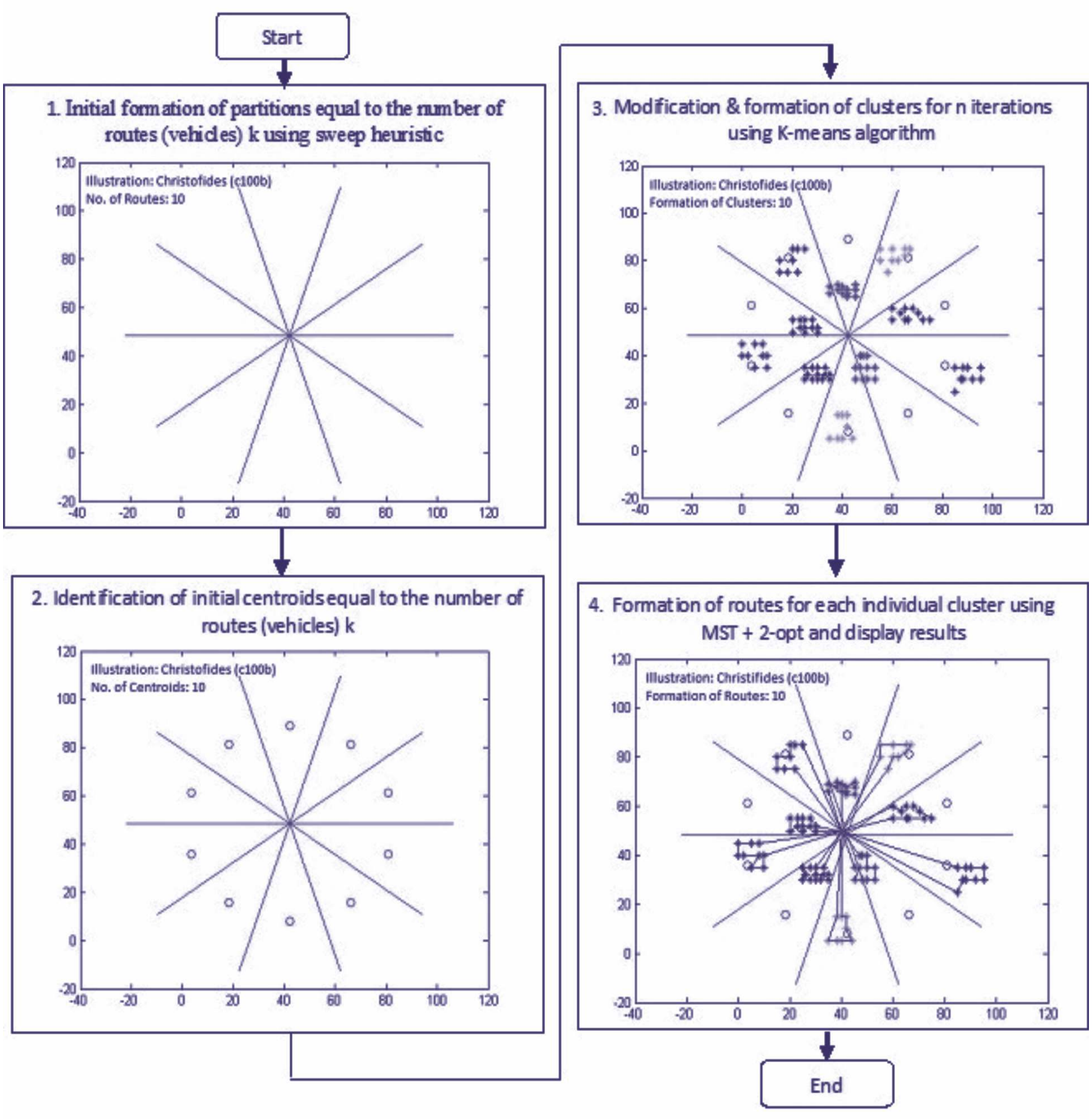

Figure 2: Illustration of Hybrid Heuristic (Christofides c100b Instance)

Sr.No.	n	Type	C&W - Sequential	C&W - Sequential + 3-opt		C&W - Parallel	C&W - Parallel + 3-opt		Sweep Algorithm	1-Petal Algorithm	2-Petal Algorithm	K-Means+MST+2Opt		Best Known Solution
			f*	f*	k	f*	f*	k	f*	f*	f*	f*	k	f*
1	50	C	625.56	624.2	5	584.64	578.56	6	531.9	531.9	524.61	537.75	5	524.61
2	75	C	1005.25	991.94	10	900.26	888.04	10	884.2	885.02	854.09	878.26	10	835.26
3	100a	C	982.48	980.93	8	886.83	878.7	8	846.34	836.34	830.4	904.13	8	826.14
4	100b	C	939.99	928.64	10	833.51	824.42	10	919.51	824.77	824.77	832	10	819.56
5	120	C	1291.33	1232.9	7	1071.07	1048.53	7	1265.65	1252.84	1109.14	1105	7	1042.11
6	150	C	1299.39	1270.34	12	1133.43	1128.24	12	1075.38	1070.5	1054.62	1155.8	12	1028.42
7	199	C	1708	1667.65	16	1395.74	1386.84	17	1396.05	1406.84	1354.23	1650.4	16	1291.45

1. n: number of vehicles
2. C: capacity constraint
3. f*: solution obtained
4. k: number of vehicles

Figure 3: Comparison of Proposed Heuristic with Classical Heuristics of Christofides Benchmark Instances

5. CONCLUSION

The paper presents a new hybrid heuristic adopting sweep, K-means clustering, minimal spanning tree and 2-opt combination for solving a multi-objective capacitated vehicle routing problem. An important finding is that cluster first and route second approaches in general have limitations if intra-route improvements are made in the improvement phase as is the case with 2-opt method. The proposed solutions in this paper can be further improved using inter route improvements. The proposed approach is based on combination of simple classical heuristics and is easy to understand and implement in comparison with meta-heuristics.

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